Free and Forced Convection in Conduits with Asymmetric Mass Transfer

WILLIAM N. GILL

Clarkson College of Technology, Potsdam, New York

EDUARDO DEL CASAL and DALE W. ZEH

Syracuse University, Syracuse, New York

Approximate solutions for the thermal entrance region and a new exact solution for the fully developed region have been obtained for nonlinear fully developed conduit flows with unsymmetrical transverse flow. Entrance region solutions are obtained by a generalized version of Mercer's method. The effects on combined free and forced convection of asymmetric mass transfer, in the asymptotic region of fully developed thermal and hydrodynamic conditions, are studied and a new exact solution is given which applies to such problems.

In practice, when secondary flows are employed for transpiration purposes or for distributing a critical reactant along a porous-walled flow reactor, these transverse flows will very likely exhibit varying degrees of asymmetry. Thus, the purpose of this work is to investigate the influence of such unsymmetrical conditions on heat and momentum transfer in conduits with fully developed velocity fields, in both the thermal entrance region where free convection effects are neglected, and in the fully developed thermal region where free convection effects are included. To do this it is necessary to use a slightly more general stream function than has been used previously for conduit flows with finite transverse velocity at the wall.

In the present work it is shown that a generalized version of Mercer's method (*4, 5*), which is an extension of Leveque's approach, can be employed to study heat transfer in the thermal entrance region of conduit flows with unsymmetrical transverse velocities. Furthermore, the procedure which has been used to study combined free and forced convection in the fully developed region (*2*) has been generalized to account for unsymmetrical conditions.

GENERAL EQUATIONS

Before considering individual cases, the general equations for the fully developed flow through two-dimensional ducts as sketched in Figure 1 will be investigated. The dimensionless equations of change are

Momentum:

$$\nabla^4\psi = \frac{\partial\psi}{\partial y_1}\cdot\frac{\partial}{\partial x_1}(\nabla^2\psi) - \frac{\partial\psi}{\partial x_1}\cdot\frac{\partial}{\partial y_1}(\nabla^2\psi) + \frac{ga^3}{\nu^2}\left[\frac{\partial}{\partial y_1}\left(\frac{\rho}{\rho_o}\sin W\right) - \frac{\partial}{\partial x_1}\left(\frac{\rho}{\rho_o}\cos W\right)\right] \quad (1)$$

Energy:

$$\left[\nabla^2 + N_{Pr}\left(\frac{\partial\psi}{\partial x_1}\cdot\frac{\partial}{\partial y_1} - \frac{\partial\psi}{\partial y_1}\cdot\frac{\partial}{\partial x_1}\right)\right] t = 0 \quad (2)$$

where

$$u = \frac{\nu}{a}\,\frac{\partial\psi}{\partial y_1}, \quad -v = \frac{\nu}{a}\,\frac{\partial\psi}{\partial x_1} \quad (3)$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2}, \quad \nabla^4 = \frac{\partial^4}{\partial x_1^4} + 2\frac{\partial^4}{\partial x_1^2\,\partial y_1^2} + \frac{\partial^4}{\partial y_1^4}$$

and W is the angle of inclination from the earth's surface. The equation of state, as usual, is taken as

$$\rho/\rho_o = 1 - \beta(t - t_o) \quad (4)$$

Equations (1), (2), (3), and (4) imply that the physical properties are constant except in the body force term, and that temperature differences in the system are not too large, so that the linear equation of state is valid.

For constant, but not necessarily equal, interfacial velocities at the conduit walls, if one defines the dimensionless stream function as

$$\psi = \frac{a}{\nu}\{[(v_{w1} - v_{w2})x_1 + U_o]\,F(y_1) - v_{w1}x_1\} = \frac{a}{\nu}[UF(y_1) - v_{w1}x_1] \quad (5)$$

which is somewhat more general than that proposed by Berman (*1*), then the momentum equation becomes

$$F''''(y_1) = \epsilon\{F'''(y_1) + (1-K)[F'(y_1)F''(y_1) - F(y_1)F'''(y_1)]\} \quad (6)$$
$$+ \frac{ga^3}{\nu^2 N_{Re}}\cdot\frac{\left[\frac{\partial}{\partial y_1}\left(\frac{\rho}{\rho_o}\sin W\right) - \frac{\partial}{\partial x_1}\left(\frac{\rho}{\rho_o}\cos W\right)\right]}{[1 + r(1-K)x_1]}$$

where

$$U = U_o + (v_{w1} - v_{w2})\,x_1, \quad r = \frac{v_{w1}}{U_o}, \quad K = \frac{v_{w2}}{v_{w1}}, \quad \epsilon = \frac{v_{w1}a}{\nu}$$

These definitions clearly require that if there is finite interfacial velocity at only one wall, then this wall must correspond to $y = 0$. Furthermore, it is assumed that the interfacial velocity of greatest absolute magnitude occurs at $y = 0$, so $-1 \leq K \leq 1$. The boundary conditions are

$$F'(1) = F'(0) = F(0) = 0$$
$$F(1) = 1 \quad (7)$$

Three properties of Equation (6) make it difficult to solve. First, it is nonlinear, and second, it is coupled with

Eduardo del Casal is with Boeing Company, Seattle, Washington. Dale W. Zeh is with Jet Propulsion Laboratory, Pasadena, California.

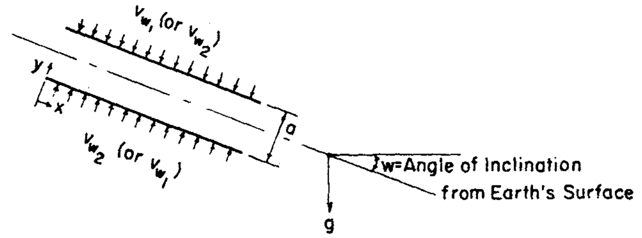

Fig. 1. Diagram of coordinate system.

the energy equation when buoyancy is not negligible. Third, two boundary conditions are specified at $y_1 = 0$ and two at $y_1 = 1$. Therefore, forward integration schemes, such as Runge Kutta, must iterate on both the boundary conditions at $y_1 = 1$ and the temperature functions as well. Of course, a significant simplification results if one employs a perturbation method, with, say ϵ as expansion parameter, to solve Equation (6). Then the system is linearized and one can solve it much more easily. However, as will be shown later, perturbation methods must be used with caution when solving the energy equation.

Specific forms of Equation (6), with negligible body forces, have been considered by a number of investigators for $K = -1$, the symmetric case, $K = 0$, the semiporous case, and the linear case when $K = 1$. The case of vertical and horizontal ducts with body forces and $K = -1$ has also been studied (2), and most of the relevant literature on combined free and forced convection in conduits with and without mass transfer is discussed in this reference.

ENTRANCE REGION RESULTS

In the entrance region, where the thermal boundary layer is thin, buoyancy effects are at a minimum for a given set of thermal and flow conditions. Thus, the case of negligible body forces will be considered to assess the importance of unsymmetrical mass transfer on entrance region heat transfer in fully developed flow fields. For this purpose let

$$u = \sum_{j=1}^{\infty} \frac{1}{j!} \frac{\partial^j u}{\partial y_1^j}(x_1, 0)\, y_1^j = U \sum_{j=1}^{\infty} \frac{1}{j!} F^{(j+1)}(0)\, y_1^j$$

where $F^{(j+1)}(0)$ denotes the $j+1$ derivative of F evaluated at $y_1 = 0$, so that by neglecting $\partial^2 t/\partial x^2$ one can write the energy equation for the lower wall region as

$$\left(\sum_{j=1}^{\infty} \frac{F^{(j+1)}(0)}{j!\,F''(0)} (\epsilon N_{Pr})^{1-j} \eta^j \right) \frac{\partial t}{\partial \xi} + \left[1 - (1-K) \sum_{j=1}^{\infty} \frac{F^{(j+1)}(0)}{(j+1)!} (\epsilon N_{Pr})^{-(j+1)} \eta^{j+1} \right] \frac{\partial t}{\partial \eta} = \frac{\partial^2 t}{\partial \eta^2} \tag{8}$$

where

$$\eta = \epsilon N_{Pr} y_1, \quad \xi = \frac{(\epsilon N_{Pr})^3}{N_{Pe} F''(0)} \int_0^{x_1} \frac{dx_1}{U/U_o} = \frac{(\epsilon N_{Pr})^3}{N_{Pe} F''(0)} \; \frac{\ln [1 + r(1-K)x_1]}{r(1-K)}$$

The development which follows can be applied to Equation (8) in its present form without difficulty, but the numerical work is simplified substantially if the assumption of a linearized velocity near the wall is employed. This is the well-known Leveque approximation which is most accurate at high N_{Pr} (6). With this assumption, Equation (8) becomes

$$\eta \frac{\partial t}{\partial \xi} + \left[1 - (1-K) \frac{F''(0)}{2} (\epsilon N_{Pr})^{-2} \eta^2 \right] \frac{\partial t}{\partial \eta} = \frac{\partial^2 t}{\partial \eta^2} \tag{9}$$

Equation (9) also applies to the upper wall region if one lets $F''(0) \rightarrow F''(1)$ and $y_1 \rightarrow Z$ where $Z = 1 - y_1$.

If

$$\sigma = (9\xi)^{1/3}, \quad \beta = \eta\sigma^{-1}$$

then Equation (9) is transformed to

$$3\sigma\beta \frac{\partial t}{\partial \sigma} = \frac{\partial^2 t}{\partial \beta^2} + \left\{ 3\beta^2 - \sigma \left[1 - \frac{F''(0)(1-K)}{2(\epsilon N_{Pr})^2} \sigma^2 \beta^2 \right] \right\} \frac{\partial t}{\partial \beta} \tag{10}$$

Since interest is primarily in the entrance region, it is reasonable to expand t in a Taylor series in σ as

$$t = \sum_{j=0}^{\infty} t_j(\beta)\, \sigma^j$$

and then to obtain the set

$$t_o'' + 3\beta^2 t_o' = 0 \tag{11a}$$

$$t_1'' + 3\beta^2 t_1' - 3\beta t_1 = t_o' \tag{11b}$$

$$t_2'' + 3\beta^2 t_2' - 6\beta t_2 = t_1' \tag{11c}$$

$$t''_{j+3} + 3\beta^2 t'_{j+3} - (j+3)\, 3\beta t_{j+3} = t'_{j+2} + (K-1)(\epsilon N_{Pr})^{-2} \frac{F''(0)}{2} \beta^2 t'_j : j = 0, 1, 2 \ldots$$

It is significant to note that t_o, t_1, and t_2 can be calculated once and for all since the parameter $\frac{(K-1)F''(0)}{2(\epsilon N_{Pr})^2} = B$ enters into the calculation only for the third order and higher terms. Furthermore, the entire calculation can be carried out once and for all either if $K = 1$, or if the term involving B is negligible compared to unity. To obtain an estimate of when this may occur, note that one can write

$$1 - \frac{F''(0)(1-K)}{2(\epsilon N_{Pr})^2} \sigma^2 \beta^2 = 1 - \frac{F''(0)\,(1-K)}{2} \left\{ \frac{9 \ln [1 + r(1-K)x_1]}{r(1-K)\,F''(0)\,N_{Pe}} \right\}^{2/3} \beta^2$$

and if $r(1-K) << 1$ this yields

$$1 - \frac{F''(0)\,(1-K)\,\sigma^2\beta^2}{2(\epsilon N_{Pr})^2} = 1 - \frac{F''(0)(1-K)}{2} \left\{ \frac{9x_1}{F''(0)\,N_{Pe}} \right\}^{2/3} \beta^2$$

From solutions of Equations (11) up to and including θ_2, it appears one can assume that the thermal boundary layer is confined approximately to $0 \leq \beta \leq 1.5$. Therefore, the term involving $F''(0)$ is negligible if

$$x_1 << \frac{F''(0)\,N_{Pe}}{9} [1.125\, F''(0)\,(1-K)]^{3/2}$$

If the wall temperature is constant it is convenient to let

$$\theta = \frac{t - t_i}{t_w - t_i} = \sum_{j=0}^{\infty} \theta_j(\beta)\, \sigma^j \tag{12}$$

Then the θ_j will satisfy Equations (11) and the boundary conditions

$$\theta_o(0) = 1, \; \theta_j(0) = 0, \; j \geq 1, \; \theta_j(\infty) = 0 \tag{13}$$

TABLE 1. VALUES OF DIMENSIONLESS VELOCITY GRADIENTS FOR THE SYMMETRIC CASE OF $K = -1$

ϵ	0.000	0.250	1.000	2.000	3.000	4.000	5.000	−0.250	−0.500	−0.750	−1.500	−2.500	−5.000
$F''(0)$	6.000	5.979	5.919	5.847	5.782	5.724	5.670	6.022	6.044	6.067	6.140	6.248	6.579

Clearly, with constant wall temperature, θ_o and θ_1 can be obtained exactly as

$$\theta_o = 1 - \frac{\int_o^\beta e^{-\beta^3} d\beta}{\int_o^\infty e^{-\beta^3} d\beta} \tag{14}$$

and

$$\theta_1 = \frac{\beta \theta_o}{2} \tag{15}$$

but the $\theta_j(\beta)$ for $j \geq 2$ must be determined by numerical integration. In this study only the solutions to θ_1 and θ_2 are presented. These results lead to

$$N_{Nu_{tw}} = \frac{ha}{k} = \frac{\epsilon N_{Pr}}{\sigma} \left\{ \frac{1}{\int_o^\infty e^{-\beta^3} d\beta} - \frac{\sigma}{2} - \sum_{j=0}^{\infty} \theta'_{j+2}(0)\, \sigma^{j+2} \right\} = \left[\frac{9F''(0)\, N_{Pe}\, r(1-K)}{\ln[1 + r(1-K)\, x_1]} \right]^{1/3} \frac{1}{\int_o^\infty e^{-\beta^3} d\beta} - \frac{\epsilon N_{Pr}}{2} - \epsilon N_{Pr} \sigma \sum_{j=0}^{\infty} \theta_{j+2}(0)\, \sigma^j \tag{16}$$

Clearly, if $K = 1$, that is, if $v_{w_1} = v_{w_2}$, the first term reduces to the Leveque solution, with the remaining terms accounting for transverse velocity effects. In this case

$$\sigma = \epsilon N_{Pr} \left[\frac{x_1}{9F''(0)\, N_{Pe}} \right]^{1/3}$$

$\theta_2'(0)$ can be calculated once and for all and was found to be equal to −0.0716.

For constant heat flux q_w at the wall, define

$$\phi = \frac{t - t_i}{q_w a / k\, \epsilon N_{Pr}} = \sum_{j=1}^{\infty} \phi_j(\beta)\, \sigma^j \tag{17}$$

where t_i is the inlet temperature. The ϕ_j will also satisfy Equations (11) and the boundary conditions are

$$\frac{\partial \phi_1(0)}{\partial \beta} = -1,\quad \frac{\partial \phi_j(0)}{\partial \beta} = 0,\ j \neq 1,\ \phi_j(\infty) = 0 \tag{18}$$

With constant heat flux $\phi_o = 0$, and therefore

$$\phi_1 = \frac{e^{-\beta^3}}{\Gamma\left(\frac{2}{3}\right)} - \beta \left\{ 1 - \frac{\Gamma\left(\frac{2}{3}, \beta^3\right)}{\Gamma\left(\frac{2}{3}\right)} \right\} \tag{19}$$

where $\Gamma\left(\frac{2}{3}, \beta^3\right)$ is the incomplete gamma function. Consequently, the Nusselt number is given by

$$N_{Nu_{qw}} = \frac{ha}{k} = -\frac{\partial\phi(0)/\partial y_1}{\phi(0)} = \frac{\epsilon N_{Pr} \sigma^{-1}}{\frac{1}{\Gamma(2/3)} + \sum_{j=0}^{\infty} \phi_{j+2}(0)\, \sigma^{j+1}} = \left[\frac{9F''(0)\, N_{Pe}\, r(1-K)}{\ln[1 + r(1-K)\, x_1]} \right]^{1/3} \left[\frac{1}{\Gamma(2/3)} + \sum_{j=0}^{\infty} \phi_{j+2}(0)\, \sigma^{j+1} \right]^{-1} \tag{20}$$

$\phi_2(0)$ involves a once and for all calculation and was found to be equal to 0.2410.

Consequently, for the cases of constant wall temperature and constant wall heat flux, solutions including second-order effects have been obtained which are valid for high Prandtl number flows. However, it is known that for the entrance region results obtained by employing a linear velocity distribution are of practical value even for $N_{Pr} = 0.7$. It is worth emphasizing that for the case of constant wall velocity being considered, $N_{Nu_{tw}}$ and $N_{Nu_{qw}}$ may be expressed, including second-order terms, as functions of only ϵ, K, N_{Pr}, $F''(0)$ and x_1. Exact values of $F''(0)$ for symmetrical mass transfer and the semiporous duct have been tabulated by Eckert, Donoughe, and Moore (3). Some supplementary data for the symmetric case are given in Table 1.

FULLY DEVELOPED THERMAL REGION RESULTS

In the region at large distances from the thermal inlet, it is possible to find functions for the temperature distribution which are consistent with Equations (2) and (6) for both vertical and horizontal conduits. This is fortunate because body forces will be most pronounced in this region where the temperature distribution is fully developed and density variations exist across the entire channel.

First, the coupled Equations (2) and (6) will be reduced to total differential equations for both vertical and horizontal conduits. Then a new closed form exact solution for horizontal conduits will be given. Finally, a limited amount of heat transfer data, based on numerical solutions to nonlinear systems, will be presented.

For horizontal systems, one obtains similar solutions if

$$t = t_{w_1} + \frac{A\, N_{Pe}\, U^2}{U_o^2}\, \theta_h(y_1) \tag{21}$$

where the lower wall temperature t_{w_1} is given as

$$t_{w_1} = t_o + A \left[x_1 + \frac{r}{2}(1-K)\, x_1^2 \right] \tag{22}$$

so that Equation (6) becomes

$$F_h'''' = \epsilon \{ F_h''' + (1-K)\, [F_h' F_h'' - F_h F_h'''] \} + N\, [1 + 2\epsilon N_{Pr} (1-K)\, \theta_h] \tag{23}$$

If one neglects dissipation and compression work, the energy equation is

$$\theta_h'' = \epsilon N_{Pr} (1-K)\, [2F_h' \theta_h - F_h \theta_h'] + \epsilon N_{Pr} \theta_h' + F_h' \tag{24}$$

The boundary conditions are

$$F_h(0) = F_h'(0) = F_h'(1) = \theta_h(0) = 0;\quad F_h(1) = 1;\ \theta_h(1) = \text{constant} \tag{25}$$

It is worth noting that Equation (21) requires the temperature difference between the two walls to be zero, or of the form

$$t_{w_2} - t_{w_1} \propto \left(\frac{U}{U_o}\right)^2 = [1 + r(1-K)x_1]^2$$

For vertical systems let

$$t = t_{w_1} + \frac{A\, N_{Pe}\, U}{U_o}\, \theta_v(y_1) \tag{26}$$

with

$$t_{w_1} = t_o + Ax_1$$

which gives

$$F_v'''' = \epsilon\, \{F_v''' + (1-K)\, [F_v'\, F_v'' - F_v\, F_v''']\} - N\, \theta_v' \tag{27}$$

and

$$\theta_v'' = (1-K)\, \epsilon\, N_{Pr}\, [F_v'\, \theta_v - F_v \theta_v'] + \epsilon N_{Pr}\, \theta_v' + F_v' \tag{28}$$

The boundary conditions are the same as those given by Equations (25). In this case the temperature difference between walls must be zero or of the form

$$t_{w_2} - t_{w_1} \propto \frac{U}{U_o} = 1 + r\,(1-K)\, x_1$$

When either $K = 1$ or $r = 0$, the temperature distribution functions for both attitudes reduce to linear wall temperature. In the fully developed region this case corresponds to constant heat flux.

As mentioned previously, specific cases relating to Equations (23) to (28) have been considered in the past. However, when these equations are written in the form used here, it is clear that an exact solution exists which has not been reported previously. This new closed form exact solution to Equations (23) to (25) for $K = 1$ can be written as

$$F_k' = \frac{[12\epsilon - N]\, [\epsilon(1 - e^{\epsilon y_1}) + (e^\epsilon - 1)\epsilon y_1] + 3N(y_1 - y_1^2)\, [(\epsilon + 2) + (\epsilon - 2)e^\epsilon]}{6\epsilon\, [(\epsilon + 2) + (\epsilon - 2)e^\epsilon]}$$

and

$$\theta_k = Q\left[\frac{e^{\epsilon\, N_{Pr} y_1} - 1}{e^{\epsilon\, N_{Pr}} - 1}\right] + \int_o^{y_1} e^{\epsilon\, N_{Pr} y_1} \int_o^{y_1} F_k'(s)\, e^{-\epsilon\, N_{Pr} s}\, ds\, dy_1$$

where

$$Q = \theta_k(1) - \int_o^1 e^{\epsilon\, N_{Pr} y_1} \int_o^{y_1} F_k'(s)\, e^{-\epsilon\, N_{Pr} s}\, ds\, dy_1$$

It can be shown that as $\epsilon \to 0$

$$F_k' \to 6(y_1 - y_1^2) + \frac{N}{24}\, (4y_1^3 - 6y_1^2 + 2y_1)$$

and as $N \to 0$

$$F_k' \to \frac{2\epsilon\, [1 + (e^\epsilon - 1)\, y_1 - e^{\epsilon y_1}]}{[(\epsilon + 2) + (\epsilon - 2)\, e^\epsilon]}$$

Friction factors f can be determined readily from

$$f(y_1 = 0) = \frac{2}{N_{Re}}\, F_k''\, (0)$$

$$f(y_1 = 1) = -\frac{2}{N_{Re}}\, F_k''\, (1)$$

for the lower and upper walls, respectively. Correspondingly, Nusselt numbers may be computed from

$$N_{Nu}\, (y_1 = 0) = \theta'\, (0)/\theta_m$$

$$N_{Nu}\, (y_1 = 1) = -\frac{\theta'\, (1)}{\theta_m - \theta\, (1)}$$

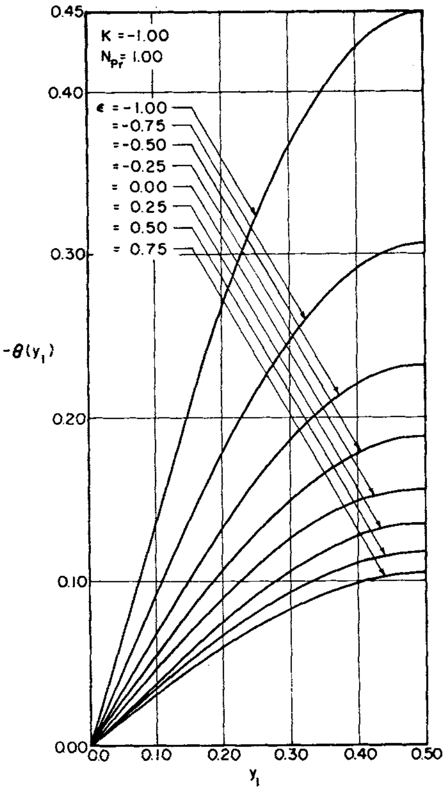

Fig. 2. Effect of the wall Reynolds number ϵ on the temperature profiles for fully developed flow with symmetrical mass transfer.

$$\theta_m = \int_o^1 \theta F'\, dy_1$$

When $K = 1$, the vertical case described by Equations (27) and (25) also can be easily solved. However, Rao (7) has already obtained exact solutions to this case for pure free convection in vertical ducts and the extension to combined free and corced convection is straightforward and need not be carried out in detail here.

Aside from the special case of $K = 1$, Equations (23) and (27) are nonlinear and cannot be solved in closed form. One must employ series expansions or numerical methods to obtain solutions. Because of the number of parameters involved, a comprehensive numerical study of Equations (23) to (28) is formidable and only a few special cases will be considered here.

In a previous study (2) approximate solutions to Equations (23) to (28) were obtained for $K = -1$, and it was

Table 2. Comparison of Nusselt Numbers Obtained from Exact and Perturbation Methods for Symmetrical Mass Transfer with $K = -1.0$ and $N_{Pr} = \cdot 1.0$

$N_{Nu_{exact}}$	$N_{Nu_{perturbation}}$	ϵ
4.344	4.567	−0.75
4.267	4.448	−0.50
4.191	4.302	−0.25
4.147	—	−0.10
4.118	4.118	0
3.839	2.359	1.0

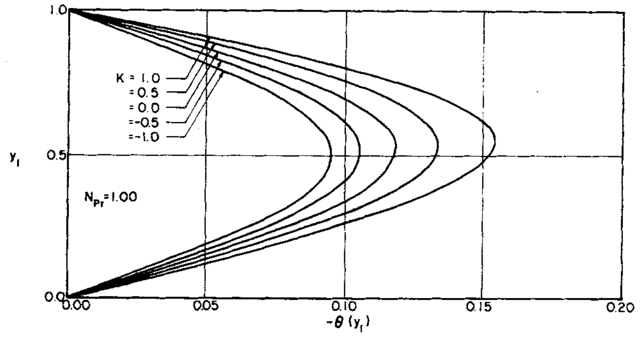

Fig. 3. Effect of asymmetric mass transfer on temperature profiles with ϵ = 1.0.

found that for comparable wall velocities, suction altered temperature distributions substantially more than blowing. This result was corroborated by the exact numerical solutions obtained in the present study as shown in Figure 2.

In the previous study (2) results were obtained by using both the method of averages and a perturbation technique. In Table 2, Nusselt numbers determined from the perturbation method are compared with exact values and is seen that the perturbation solutions are more accurate for cases involving suction than blowing. In contrast, the method of averages predicts Nusselt numbers very well in systems with blowing. For example, with N_{Pr} = 1.0, ϵ = 1.0, K = — 1.0, the Nusselt numbers obtained by the exact, average, and perturbation methods are 3.839, 3.783, and 2.359.

The effect of asymmetric mass transfer on fully developed temperature profiles is shown in Figure 3 for various values of K. A positive value of K indicates that the two wall velocities are in the same direction, that is, blowing at one wall and suction at the other. On the other hand, negative values indicate that the wall velocities are in opposite directions. Clearly, the profiles become more unsymmetrical as one proceeds from K = — 1.0 to K = 1.0. Since $\epsilon > 0$ for all cases shown, execpt for K = 1.0, the mass flow rate increases in the direction of flow which, on the basis of an overall mass balance, might be considered qualitatively similar to the symmetric case with a modified blowing rate. Thus, one can expect an overall depression of the temperature profiles as is observed in Figure 3.

Figure 4 illustrates the behavior of Nusselt numbers at upper and lower walls as K varies from —1.0 to 1.0. As the degree of asymmetry increases for constant wall Reynolds number, the mean Nusselt number for the two walls increases. This occurs because the Nusselt number at the upper wall increases more rapidly than that at the lower

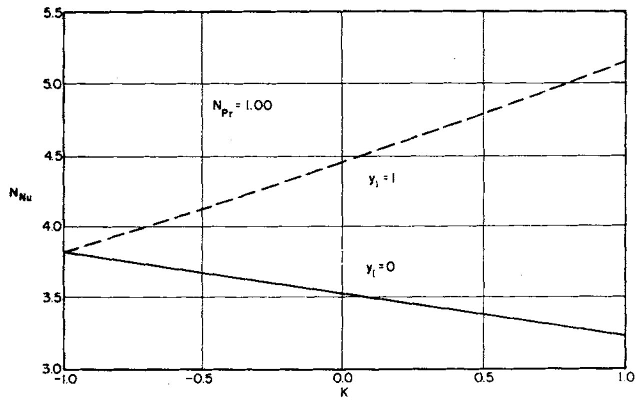

Fig. 4. Effect of asymmetric mass transfer on Nusselt numbers at upper and lower walls with ϵ = 1.00.

TABLE 3. EXACT NUSSELT NUMBERS AT UPPER AND LOWER WALLS FOR UNSYMMETRICAL MASS TRANSFER WITH ϵ = 1.0 AND N_{Pr} = 1.0

$N_{Nu}(y_1 = 0)$	$N_{Nu}(y_1 = 1)$	K
3.839	3.839	—1.0
4.134	3.689	—0.5
4.452	3.539	0
4.794	3.390	0.5
5.163	3.231	1.0

wall decreases. In the case shown, although the mass flow rate increases with x, the mean Nusselt number exceeds that for the case of no transverse flow when $K \geqq 0.65$. Some exact values of the Nusselt numbers at y_1 = 0 and y_1 = 1, for unsymmetrical mass transfer conditions are tabulated in Table 3.

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NOTATION

A defined by $dt_{w1}/dx_1 = (U/U_o)A$ for horizontal systems and $dt_{w1}/dx_1 = A$ for vertical systems
a = distance between plates
B = $\dfrac{(K-1)F''(0)}{2(\epsilon N_{Pr})^2}$
g = gravitational constant
K = ratio of wall velocity at y = 1 to wall velocity at y = 0
k = thermal conductivity
N = N_{Gr}/N_{Re}
$N_{Nu_{qw}}$ = Nusselt number for constant heat flux boundary condition
$N_{Nu_{tw}}$ = Nusselt number for constant wall temperature boundary condition
N_{Gr} = Grashof number $\beta_1 g A a^3/\nu^2$
N_{Pe} = Peclet number, $U_o a/\alpha$
N_{Pr} = Prandtl number, ν/α
N_{Re} = Ua/ν
q_w = wall heat flux
t = temperature
t_i = inlet temperature in entrance region case
t_o = reference temperature associated with x_1 = 0 for fully developed case
t_w = wall temperature
t_{w_1} = wall temperature at y_1 = 0 for fully developed case
t_{w_2} = wall temperature at y_1 = 1 for fully developed case
U = bulk velocity at any x_1
U_o = reference bulk velocity
u = local axial velocity
v = local transverse velocity
v_{w_1} = transverse velocity at y_1 = 0
v_{w_2} = transverse velocity at y_1 = 1
x = axial distance
x_1 = reduced axial distance, x/a
y = transverse distance
y_1 = reduced transverse distance, y/a

Greek Letters

α = $\dfrac{k}{\rho Cp}$
β = dimensionless coordinate defined in text

β_1 = volumetric expansion coefficient
Γ = gamma function
θ_m = dimensionless bulk mean temperature
θ_h = dimensionless temperature function for horizontal systems
θ_v = dimensionless temperature function for vertical systems
ν = kinematic viscosity
ρ = density
ρ_o = reference density
ψ = dimensionless stream function

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Simultaneous Axial Dispersion and Adsorption in a Packed Bed

RAUL CHAO and H. E. HOELSCHER

Johns Hopkins University, Baltimore, Maryland

A study has been made of simultaneous axial dispersion and solid-fluid mass exchange in a packed-bed adsorber. Four different and mutually exclusive controlling mechanisms for the solid-fluid mass exchange rate are considered. A dimensionless parameter *K* characterizes this interphase mass transfer. The Peclet number characterizes axial dispersion.

An impulse-response technique was used to obtain simultaneously values of the axial Peclet number and the rate parameter *K* in the adsorption column. Values of the Peclet number obtained under conditions of interphase mass transfer were found to be significantly smaller than the values measured under pure mixing (no surface activity) conditions.

The mathematical model used to analyze the results includes the particular case of no surface activity with results previously found from the dispersion model. One result not previously derived from the dispersion model was found and tested experimentally.

Recently, considerable attention has been focussed on systems in which chemical processes as well as heat and/or mass transfer take place simultaneously. A complete description of such systems is available for only the simplest geometries, but an increased interest in transport phenomena in, for example, beds of porous solids, is more than justified by the demand for more rigorous design of catalytic reactors, chromatographic and ion exchange columns, beds of adsorbents, etc.

The design of a catalytic reactor must be based on the continuity, energy, and momentum equations coupled by a common term containing the reaction rate. A frontal approach to this set of differential equations remains prohibitively and hopelessly complicated. It is recognized, however, that an understanding of those problems which involve only mass transfer serves as a basis for an understanding of similar problems which are complicated by the presence of chemical reactions. Thus adsorbent beds or chromatographic columns are the natural precursors of packed bed reactors for studies of dynamic responses of the latter.

In any such study, one must consider mixing processes, interfacial mass transfer, surface and pore diffusion, and adsorption processes as well as the chemical reactions that may occur.

Analyses of longitudinal and radial interparticle diffusion by Aris and Amundson (*3*), Carberry and Bretton (*7*), McHenry and Wilhelm (*23*), and Wehner and Wilhelm (*25*) have grown into a vast literature, mostly utilizing a dispersion model (diffusion type of equation with a diffusivitylike coefficient). These efforts, however,

Raul Chao is with Esso Research and Engineering Company, Florham Park, New Jersey. H. E. Hoelscher is at the University of Pittsburgh, Pittsburgh, Pennylvania.